Math Aversion

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Students in my Basic Electronic Design course had a difficult time distinguishing between the graphical representations of $e^{-t/\tau}$ and $(1- e^{-t/\tau})$. They had no gut feeling for either term; they didn't really know that the first decreases with time asymptotically to zero, but the second increases with time asymptotically to 1. These are very basic functions to electronics, chemistry, physics, and biology, and knowing what they mean is important to understanding of the world as it is. It took many quiz questions, and many wrong answers, before the majority of students could identify one from the other. Some students just memorized the answers.

Moving on to transfer functions, these juniors, who had completed three calculus courses plus differential equations, were flummoxed by the meanings of the equations; they had seen many of the terms before, but had no facility to visualize what they actually stood for. Many of them couldn't comprehend that the equations represented actual responses in the frequency and time domains because they couldn't translate from one to another. It took laboratory exercises to demonstrate to them what they could not comprehend in lecture. Again, some resorted to memorization.

Students in my Transport Process Design course would do anything they could to avoid having to deal with simple differential equations. We are talking here about first and second order differential equations with constant coefficients; they were not that complicated. Start talking about any calculus math at all, and students' eyes would begin to glaze.

I could relate somewhat to this attitude. When I was in high school, mathematics fascinated me (so did history, but that's not relevant here). I could calculate numbers easily in my head (a largely forgotten talent), but when I moved on to college, things started going a little flat. I was put in an advanced-level introductory calculus course, but it proved beyond me at the time. Once I dropped back to the normal-level course, things again righted themselves.

In those days, we engineering students were proud to walk across campus with our slide rule cases swinging to and fro from our belts. Those were the days before handy-dandy calculators, and we lived and died (not literally) by our slide rule prowess. We learned how to estimate numerical answers in our heads so that we knew where to place the decimal points in the answers that came up on our slide rules. Pi could be approximated by 3.0, and factors in the numerator were canceled with similar, but not identical, factors in the denominator. I became rather proficient at this, and still retain enough of this ability that I can sometimes use it to impress my calculator-dependent students, especially when I can approximate an answer before they can key in all their numbers.

I loved differential equations. There were rules and strategies that one could follow to find solutions of many of the more common types of differential equations, and I really liked the order and structure of the process. Because I liked this course, I developed a facility to solve these fun equations, even finding trigonometric substitutions and integration by parts relatively easily within my grasp. I disdained my fellow students who had to use integral look-up tables.

Things started turning for the worse when I took a statistics course in the math department and the only way that statistical tests were introduced was with equations; there was no reality to the course, no explanation of what these tests meant and how they could be used, and I sought out other mathematics majors in the course to inquire if they knew what it all meant; they couldn't tell me. Later, as a graduate student, I took advanced calculus with its Green's functions, kernels, and tensors; these baffled me, because I had no idea about where they could be used or why I should be studying them. In advanced heat transfer, with its Bessel functions, error functions, and Legendre polynomials, I longed for some basic and real understanding. I could not visualize how these higher-level monsters behaved, so they remained foreign to me. I started using look-up tables.

Skip now to a time when I had come to be a university professor. Two lessons occurred that helped me understand how mathematics is really practiced. Once, when I encountered a differential equation unfamiliar to me, I had to go to a fellow faculty member, a math professor, for help. I had tried to bone up on my rules and strategies for solving differential equations, but the solution to this equation still eluded me. Instead of following the ordered procedure toward a solution that I had learned many years before, he proceeded to guess at the answer and worked backward to see if his guess satisfied the equation that I had brought to him. I learned from this experience that familiarity with mathematical solutions made finding new solutions much easier. This mathematics expert was operating just like my engineering students when they worked backward from the answers to solve their homework problems. From that point on, when faced with a new mathematical modeling problem, I tried to start with a simplified permutation of the problem, one that I understood and could be confident of the correctness of the solution, and then add complexity one step at a time.

The second lesson came up when composing a worked example to include in my second book, *Biological Process Engineering: An Analogical Approach to Fluid Flow, Heat Transfer, and Mass Transfer in Biological Systems.* The problem involved conduction with temperature-dependent heat production, but the temperature-dependent part threw me. Anyone knows that the rate of biological heat production depends on temperature, so it was important to me to include this problem in the book.

Following the procedure learned from lesson number one, I simplified the problem and found answers that I was confident were correct. However, I couldn't get around the temperature-dependence difficulty. After many tries, I gave up, walked down to the math department, and consulted a professor there. He listened as I explained the problem, and once he understood what it was that I was asking, he did something I was not prepared for; he tried to look up the solution in *Mathematica*. Not finding the solution there, he declared that he could not help me. I left his office, disappointed that it had come to this, when even the math professors had to use look-up tables. I included this example in my book anyway, but it has no solution given there.

With the insights that my own experiences had given me, I had sympathy for my students with math-aversion. I have firmly convinced myself that it is a mistake to introduce an engineering subject purely mathematically. Before they have to deal with the strange language of mathematics, they need first to understand the concepts important to the subject. They need to develop the ability to visualize, if possible, or at least understand to a fundamental degree mechanisms or ways that things work. After that, mathematical descriptions can be taught as correlates to the basic concepts. Equations are just one possible description of the ideas making a product or process happen, although an efficient description at that. With this two-step approach, students can start to understand better both the basic concept and the mathematics.

For that reason, I have tried to incorporate conceptual thinking as the first step in my courses and in my books. You will see in *Biological Process Engineering*, that I have relied on a very visual conceptual approach to transport processes, and in *Biology for Engineers* that the mathematics is kept to a minimum. Engineers need intuition to invent new solutions, but they also need mathematical discipline to assure that these solutions are realizable. There is no substitute for either.